Interactions with Granular Material

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Introduction

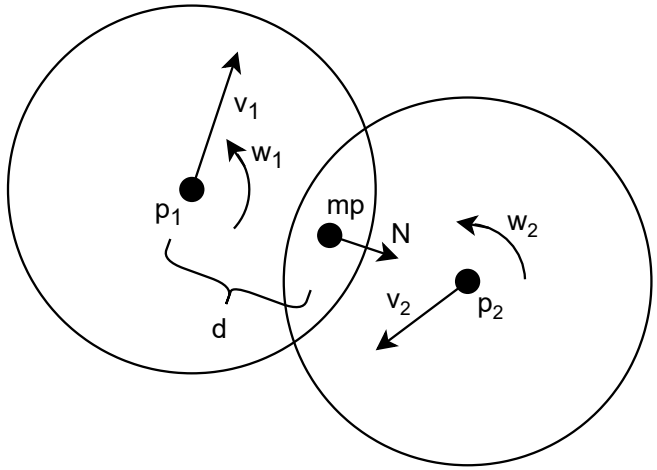
- Simulation of granular materials is important for many reasons
  - Industrial processes, graphics, geology, etc
- Simulations generally take the following approaches:
  - *Eulerian grid*-based approaches treat space as a grid of cells, similar to fluid dynamics solvers
    - ★ Key disadvantages: grid structure doesn't preserve matter; inter-granular forces are difficult to model in this paradigm
  - *Lagrangian particle*-based approaches simulate each individual particle
    - ★ Key disadvantage: computational complexity grows linearly in the number of particles
  - *Hybrid* approaches, such as *Particle-In-Cell* (PIC), attempt to merge the two
    - ★ Key disadvantages: energy dissipation, instability
    - ★ Key paper: "The Affine Particle-In-Cell Method" [1]

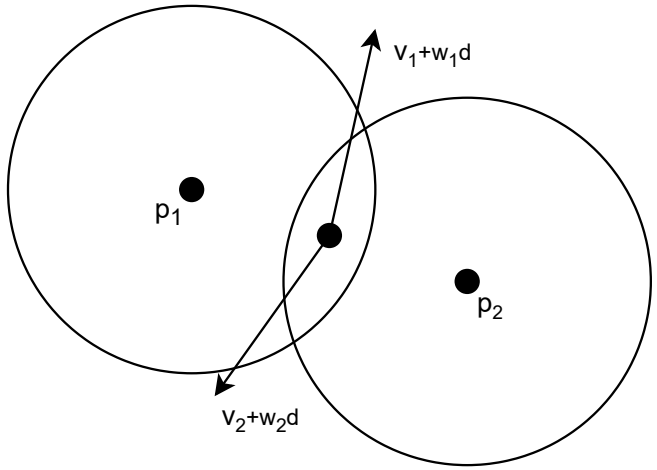
Particle Types

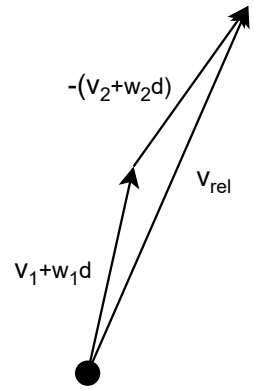
- Event-Driven (ED)
  - Can accurately and efficiently simulate continuous particle motion
  - Properly models particle rigidity and exchange of momentum/energy in collisions
  - Efficiency decreases when:
    - ★ Simultaneous collisions occur between more than two bodies
    - ★ Collisions are common and contact is prolonged
  - Unsuitable for sand simulation due to the formation of static piles
- Molecular Dynamics (MD)
  - Force-based approach that allows slight particle overlap
  - Simple to program and extend
  - Downsides:
    - ★ Requires extremely small timesteps to enforce rigidity, otherwise particles will compress
    - ★ Significantly slower than ED
  - Approach taken in "Particle-Based Simulation of Granular Materials" [2]

Circle Collision Description

- p_1, p_2: Circle centers
- v_1, v_2: Linear velocities
- w_1, w_2: Angular velocities
- mp: Collision midpoint
- N: "Line of centers" (normalized $p_2 - p_1$)
- d: Distance from center to collision (lever arm length)
- $w_1 d, w_2 d$: shorthand for rotational velocity d units away from center with angular velocity w (at mp)
- v_{rel}: relative velocity at mp







- Force calculation uses amount of overlap $\xi = 2(r - d)$ and v_{rel}

A Third Approach

- Physics engines work similarly to ED, but with discrete timesteps
- Rather than solving each collision as an event, they iteratively resolve constraints (such as non-interpenetration) on each frame
- An excellent short series on physics simulation is "Physically Based Modeling" [3]

Live Demo

Live demo of four approaches:

- Single-particle MD
- Single-particle rigid
- Polysphere MD
- "Bouncy" rigid polyspheres

Characteristics to look for:

- (In)compressibility
- Nonzero Angle of Repose
- Stability/Oscillations
- *Anisotropic force chains*

Further Reading


C. Jiang, C. Schroeder, A. Selle, J. Teran, and A. Stomakhin, "The affine particle-in-cell method," *ACM Trans. Graph.*, vol. 34, no. 4, Jul. 2015. [Online]. Available: https://doi.org/10.1145/2766996


N. Bell, Y. Yu, and P. Mucha, "Particle-based simulation of granular material," 01 2005, pp. 77–86.


A. Witkin and D. Baraff, "Physically based modeling," Available at https://graphics.pixar.com/pbm2001/ (2001).